Homework 10: 11/21 - 11/23

STA 335 Due 11:59pm Wednesday, November 30 Name:

Instructions: Write-up complete solutions to the following problems and submit answers on Gradescope. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A rubric for homework problems appears on the final page of this assignment.

- Unless otherwise noted, problem numbers are taken from the 2nd edition of Blitzstein and Hwang's *Intro to Probability*.

Monday 11/21

Chapter 7

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Additional Problem

AP1. Let $X \sim \text{Gamma}(a, \lambda)$ and $Y \sim \text{Gamma}(b, \lambda)$ be independent. Show that $X + Y \sim \text{Gamma}(a + b, \lambda)$ in two ways:

(a) Using MGFs.

(b) Using a story about a Poisson process. (For this part, assume that a and b are both positive integers).

AP2. One of the more frequently occurring variables in statistical inference is the **chi-squared** variable. If $X_1, \ldots, X_n$ are iid $N(0, 1)$, then $V_n = X_1^2 + X_2^2 + \cdots + x_n^2$ is said to have the chi-squared distribution with n degrees of freedom.

(a) Show that if V_1 is chi-squared with 1 degree of freedom, then $V_1 \sim \text{Gamma}(\frac{1}{2}, \frac{1}{2})$. *Hint: First compute the CDF of V_1 (paying special attention to $\pm$ square roots), and then differentiate to get the PDF.*

(b) Show that if V_n is chi-squared with n degrees of freedom, then $V_n \sim \text{Gamma}(\frac{n}{2}, \frac{1}{2})$.

Wednesday 11/23

Chapter 8

30, 32

Additional Problem

AP3. Suppose $U \sim \text{Unif}(0, 1)$ and let $Y = U^{1/a}$ for some $a > 0$. Show that $Y \sim \text{Beta}(a, 1)$.

AP4. As in AP1, let $X \sim \text{Gamma}(a, \lambda)$ and $Y \sim \text{Gamma}(b, \lambda)$ be independent. In this problem, you give a third way of showing that $X + Y \sim \text{Gamma}(a + b, \lambda)$.

(a) Let $Z = X + Y$. Find the conditional CDF $P(Z \leq z|X = x)$ of Z given $X = x$. Then differentiate to find the conditional CDF $f_{Z|X}(z|x)$ of Z given $X = x$. Specify the support of the CDF and PDF.

(b) Use continuous LotP (Theorem 7.1.18) to express the marginal PDF of Z in terms of the conditional PDF of Z given $X = x$ and the marginal PDF of X.

(c) Evaluate the integral by recognizing it as a Beta integral (you may need to make an appropriate u-substitution before you do so).

(d) Verify that the marginal PDF of Z is indeed the PDF for $\text{Gamma}(a + b, \lambda)$.

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General Rubric

Points	Criteria
5	The solution is correct **and** well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information)
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.