

## Simpson's Paradox

In early August 2021, Public Health England released national statistics on the Delta variant of COVID-19, reporting both the number of positive cases and deaths, classified by vaccination status. Some of this data is summarized in the table below:

Group	Vaccinated	Unvaccinated
Cases	1,173,115	1,513,054
Deaths	481	253
Mortality Rate	0.41%	0.17%

1. Does this provide evidence that the vaccine actually **increases** likelihood of mortality?
2. What are some possible explanations for this pattern?

The report also included data on the age group of cases.

Under 50		
Group	Vaccinated, < 50	Unvaccinated, < 50
Cases	893,807	1,473,612
Deaths	21	48
Mortality Rate	0.02%	0.03%

Over 50		
Group	Vaccinated, $\geq 50$	Unvaccinated, $\geq 50$
Cases	27,307	3,440
Deaths	460	205
Mortality Rate	1.68%	5.96%

This is an example of **Simpson's Paradox**:

$$A = \text{Death} \quad B = \text{Vaccinated} \quad C = \text{Over 50}$$

Then

$$P(A|B, C) < P(A|B^c, C) \quad \text{and} \quad P(A|B, C^c) < P(A|B^c, C^c)$$

But

$$P(A|B) > P(A|B^c)$$