

I. Pre-class material Either read the indicated textbook sections OR watch the indicated video.

- (a) **Sections to Read** (All content from Blitzstein and Hwang's *Introduction to Probability* unless otherwise noted). A digital copy of the textbook is available for free via the authors' website.
- 10.1 (focus just on section 10.1.3 and onward)
 - 10.2
- (b) **Videos to Watch** (All videos from Blitzstein's Math 110 YouTube channel, unless otherwise noted)
- Lecture 28: Inequalities (from 35:00 to end)
 - Lecture 29: Law of Large Numbers and Central Limit Theorem (from beginning to 15:00).

II. Objectives (By the end of the day's class, students should be able to do the following:)

- Prove the Markov inequality. Then derive the Chebyshev and Chernoff inequalities from Markov's inequality.
- Apply the Markov, Chebyshev, and Chernoff inequalities to find upper bounds on probabilities for random variables.
- State both the strong and the weak versions of the Law of Large Numbers.
- Prove the Weak Law of Large Numbers in the case when X has finite variance.
- Use the Monte Carlo method of integration to approximate definite integrals.

III. Reflection Questions (Submit answers on Gradescope <https://www.gradescope.com/courses/425901>)

- 1) What is one circumstance where the Markov inequality can be used to estimate $P(|X| > a)$, but where the Chebyshev inequality cannot be used. *Hint: Think about what the assumptions made for the Chebyshev inequality, and compare to the assumptions made for the Markov inequality.*
- 2) In one or two sentences, summarize in your own words what the Law of Large Numbers means.
- 3) Describe how you could use Monte Carlo Integration in order to approximate the area of a circle of radius 1 by generating iid uniform points in the square in the xy -plane with $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

IV. Additional Feedback Are there any topics you would like further clarification about? Do you have any additional questions based on the readings / videos? *If not, you may leave this section blank.*