

I. **Pre-class material** Either read the indicated textbook sections OR watch the indicated video.

- (a) **Sections to Read** (All content from Blitzstein and Hwang's *Introduction to Probability* unless otherwise noted). A digital copy of the textbook is available for free via the authors' website.
- 8.4 (skip the Story 8.4.5, which we'll look at after discussing 8.3 next class)
- (b) **Videos to Watch** (All videos from Blitzstein's Math 110 YouTube channel, unless otherwise noted)
- Lecture 24: Gamma Distribution and Poisson Process

II. **Objectives** (By the end of the day's class, students should be able to do the following:)

- State the integral definition of the gamma function.
- State the PDF for the Gamma distribution with parameters a and λ , and describe the shape of the distribution for various values of these parameters.
- Compute the mean, variance, and other moments of the Gamma distribution by pattern recognition.
- Show that the Gamma distribution is the conjugate prior of the Poisson distribution.

III. **Reflection Questions** (Submit answers on Gradescope <https://www.gradescope.com/courses/425901>)

- 1) Suppose a and b are positive real numbers. Use properties of the Gamma function to show the following (without doing ANY integrals):

$$\frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)} \cdot \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a+b+1)(a+b)}{ab}$$

Do not assume that a and b are integers.

- 2) Suppose the number of hours between the arrival of consecutive emails in my inbox is $\text{Expo}(3)$, independent of other waiting times. What is the distribution for the amount of time I need to wait until I have 3 emails in my inbox? What is the expected wait time?
- 3) Let t be a fixed positive real number and let j be a fixed positive integer. Suppose $X \sim \text{Pois}(t)$ and that $Y \sim \text{Gamma}(j, \lambda)$. Using a story about a Poisson process, explain why

$$P(X \geq j) = P(Y \leq t).$$

IV. **Additional Feedback** Are there any topics you would like further clarification about? Do you have any additional questions based on the readings / videos? *If not, you may leave this section blank.*