

I. **Pre-class material** Either read the indicated textbook sections OR watch the indicated video.

- (a) **Sections to Read** (All content from Blitzstein and Hwang's *Introduction to Probability* unless otherwise noted). A digital copy of the textbook is available for free via the authors' website.
- 8.1 (skip Example 8.1.10 for now; we'll revisit it after discussing 7.5 in a few weeks)
- (b) **Videos to Watch** (All videos from Blitzstein's Math 110 YouTube channel, unless otherwise noted)
- Lecture 22: Transformations and Convolutions (from 7:00 to 27:00)
 - Skim section 8.1 in the text (there is a lot of information in the text that isn't in the video)

II. **Objectives** (By the end of the day's class, students should be able to do the following:)

- State the change-of-variables formula in both 1 and several dimensions.
- Compute the PDF of the Log-Normal, the Chi-Square distribution and the Cauchy distribution using the single-variable change-of-variables formula
- Use the multivariate change-of-variables formula to compute the joint PDF of transformations of 2 or 3 variables.

III. **Reflection Questions** (Submit answers on Gradescope <https://www.gradescope.com/courses/425901>)

- 1) Suppose $y = x^3$. Compute $\frac{dx}{dy}$ as a function of y in two ways (your final answer for each should be the same):
- i. By solving for x explicitly and differentiating.
 - ii. By calculating $\frac{dy}{dx}$, taking the reciprocal, and then substituting $x = \sqrt[3]{y}$ in the final formula.
- 2) Suppose U, T are random variables with joint PDF $f_{U,T}$, and define $X = U + T$ and $Y = U - T$.
- i. Express U and T as explicit functions of X, Y .
 - ii. Calculate the Jacobian matrix $\frac{\partial(x,y)}{\partial(u,t)}$, as well as the absolute value of its determinant.
 - iii. Find a formula for the joint PDF $f_{X,Y}(x,y)$ of (X,Y) in terms of $f_{U,T}$ and the variables x,y .

IV. **Additional Feedback** Are there any topics you would like further clarification about? Do you have any additional questions based on the readings / videos? *If not, you may leave this section blank.*