Berkson's Paradox

1. (*) Suppose that in the population of college applicants, being good at baseball is independent of having a good math score on a certain standardized test (with respect to some measure of "good"). A certain college has a simple admissions procedure: admit an applicant if and only if the applicant is good at baseball or has a good math score on the test.

   (a) Give an intuitive explanation of why it makes sense that among students that the college admits, having a good math score is negatively associated with being good at baseball, i.e., conditioning on having a good math score decreases the chance of being good at baseball.

   (b) Show that if A and B are independent and $C = A \cup B$, then A and B are conditionally dependent given C (as long $P(A \cap B) > 0$ and $P(A \cup B) < 1$) with

   $$P(A|B, C) < P(A|C)$$

   This phenomenon is known as *Berkson's paradox*, especially in the context of admissions to a school, hospital, etc.

Two Dice

2. The following questions are inspired by Martin Gardner's "two children" paradox discussed in Section 2.2.

   (a) Consider an experiment where two dice are rolled (one red and one blue)

      i. What is the probability that both dice are 6?
      ii. Suppose the blue die is a 6. What is the probability that the red die is also a 6?
      iii. Suppose at least one of the dice is a 6. What is the probability that the other is a 6?
      iv. What is the fundamental difference between these three problems?

   (b) Consider an experiment where the red and blue dice are rolled, after which a fair coin is flipped. On a heads, the result of the red die is written on a slip of paper, while on a tails, the result of the blue die is written.

      i. Suppose a 6 is written on the slip of paper. What is the probability that the result of the die not written on the paper is also a 6?
      ii. How is this problem different than problem a(iii)?

   (c) Consider an experiment where the red and blue dice are rolled. Find the probability that both dice result in a 6, given that at least one is a 6 with an edge pointing in a direction between north and northwest.

   (d) By specifying a range of angles for an edge of one of the dice (as in the previous problem), describe an experiment where there is a 10% probability that both dice are 6, given that at least one is a 6 with an edge pointing in the given range.

   (e) Suppose the edge of a die falls in a certain range of angles with probability p. Find a formula (in terms of p) for the probability that both dice are 6, given that at least one is a 6 with an edge pointing in the given range? What happens as $p \to 0$? As $p \to 1$?