1. We often abbreviate the expression $E[X^2]$ as EX^2. But care should be taken to distinguish this from $(E[X])^2$. Show that $E[X^2]$ and $(E[X])^2$ are not in general equal by computing both values for $X \sim \text{DUnif}\{-1, 1\}$ (i.e. X takes the value ± 1 with equal probability).

2. (*) Player A chooses a random integer between 1 and 100, with probability p_j of choosing j (for $j = 1, 2, \ldots, 100$). Player B guesses the number that player A picked, and receives from player A that amount of dollars if the guess is correct (i.e. if player A picks 10 and player B guesses correctly, then player B gets $10).

   (a) Suppose for this part that player B knows the values of p_j. What is player B's optimal strategy to maximize **expected** earnings? *Hint: Player B does not need to make a random decision.*

   (b) Show that if both players choose their numbers so that the probability of picking j is proportional to $1/j$, then neither player has an incentive to change strategies, assuming the opponent's strategy is fixed. (In game theory terminology, this says that we have found a *Nash equilibrium*). *Note: Saying that the probability of picking j is proportional to $1/j$ means that the probability is of the form c/j for a constant c that doens't depend on j.*

   Hint: Let B be the amount of money won by Player B and assume that Player A uses the given strategy. Compute $E[B]$ and show that it doesn't depend on the strategy that player B uses.

   (c) Suppose player B follows the strategy outlined in the previous part. Find this player's expected earnings. Express your answer both as a sum of simple terms, and as a numerical approximation. Does the value depend on the strategy that player A uses?