CDFs, Transformations, and Independence

1. Use the `plot` function in R (or `ggplot`, if you are familiar with it) to plot the CDF of a $\text{Bin}(10, 0.5)$ variable. Explain how to use the values displayed in the CDF plot to recover the values of the PMF of the variable. Then verify that your method works by plotting the PMF of a $\text{Bin}(10, 0.5)$ variable.

2. (*) Let X be a discrete random variable with PMF $p(n) = \frac{1}{2^{n+1}}$ for integers $n \geq 0$, and let $Y = 2^X$. What is the support of Y? Find a formula for the pmf of Y and compute $P(Y = 1)$.

3. For x and y binary digits (0 or 1), let $x \oplus y$ be 0 if $x = y$ and 1 if $x \neq y$ (this operation is called the *exclusive or*, abbreviated to XOR, as well as *addition mod 2*).

   (a) Let $X \sim \text{Bern}(p)$ and $Y \sim \text{Bern}(1/2)$, independently. What is the distribution of the variable $Z = X \oplus Y$? Does your answer depend on p?

   (b) With the notation as in (a), is Z independent of X? Is Z independent of Y? Be sure to consider both the case $p = 1/2$ and $p \neq 1/2$ separately.