Random Variable Zoo

1. Recall de Montmort's matching problem from Chapter 1: in a deck of n cards labeled 1 through n, a match occurs when the number on the card matches the card's position in the deck. Let X denote the number of matching cards.

   (a) What is the support of X? *Hint: Think carefully about whether it is possible to have exactly $n-1$ matches.*

   (b) Is X Discrete Uniform? Bernoulli? Binomial? Hypergeometric? None of these? Explain.

2. (*) Once upon a time, a famous statistician offered tea to a lady. The lady claimed that she could tell whether milk had been added to the cup before or after the tea. The statistician decided to run some experiments to test her claim.

   (a) The lady is given 6 cups of tea, where it is known in advance that 3 will be milk-first and 3 will be tea-first, in a completely random order. The lady gets to taste each and then guess which 3 were milk-first. Assume for this part that she has no ability whatsoever to distinguish milk-first from tea-first cups of tea. Find the probability that at least 2 of her 3 guesses are correct. Then find the probability that all 3 of her guesses are correct.

   (b) Now the lady is given one cup of tea, with probability $1/2$ of it being milk-first. She needs to say whether she thinks it was milk-first. Let p_1 be the lady's probability of being correct given that it was milk first, and p_2 be the probability of being correct given that it was tea-first. She claims that the cup was milk first. Find the *posterior odds* that the cup is milk-first, given this information.

3. A national poll is conducted to estimate the U.S. president's approval rating. The polling firm randomly selects 100 U.S. voters and asks each the question "Do you approve or disapprove of the way Joe Biden is handling his job as president?". Suppose the polling firm samples *with replacement* from the population of U.S. voters, meaning that it is possible for a person to be selected for the sample more than once (in which case, their response will be recorded as many times as they appear in the sample). Let p denote the true proportion of U.S. voters who approve of the president.

   (a) What is the probability that the first person chosen for the sample approves of the president? Give your answer in terms of p.

   (b) What is the probability that at least 50 people in the sample approve of the president? Give your answer in terms of p.

   (c) Let X_1 be the random variable that takes the value 1 if the first person in the sample approves of the president, and that takes the value 0 otherwise. What is the name of the distribution of X_1? Be sure to specify the value of the parameter for this distribution as well.

   (d) Similarly, for each $k = 2, 3, \ldots, 100$, let X_k be the variable that takes the value 1 if the kth person in the sample approves of the president. What is the name of the distribution of X_k?

   (e) Let $N = X_1 + X_2 + \cdots + X_{100}$. What random quantity does the variable N represent in this context? What is the name of the distribution of N?