

1. The **frequentist** definition of the probability of an event A occurring is that it is the long-run proportion of times the event occurs in a sequence of independent trials. In other words,

$$P_{\text{freq}}(A) = \lim_{n \rightarrow \infty} \frac{I_1 + \cdots + I_n}{n}$$

where I_k is the indicator that event A occurred in the k th trial. Use the Law of Large Numbers (and the Fundamental Bridge!) to prove that the frequentist definition of probability is equivalent to the axiomatic definition we've been using throughout the term.

2. Suppose $X \sim \text{Expo}(1)$. Use each of the Markov, Chebyshev, and Chernoff inequalities to give upper bounds on the probability that X is more than 3 standard deviation above its mean. Then compute the actual probability of this event. How do each of your bounds compare to the true value?