

2D LOTUS and Covariance

1. A random point is chosen uniformly in the unit disk $\{(x, y) : x^2 + y^2 \leq 1\}$. Let R be its distance from the origin.
 - (a) Find $E[R]$ using 2D LOTUS.
 - (b) Find the CDF of R^2 and R without integrating, using the fact that the probability that the randomly chosen point is in a particular region is proportional to the area of the region.
 - (c) Then obtain the PDFs of R and R^2 by differentiating, and then use the results to calculate $E[R]$ in two more ways: by using the definition of expectation on R , and by using 1D LOTUS and thinking of R as a function of R^2 .
2. (*) Let X and Y be iid $\text{Unif}(0, 1)$.
 - (a) Compute covariance of $X + Y$ and $X - Y$.
 - (b) Are $X + Y$ and $X - Y$ independent? Explain.