1. (*) Let $X_1, X_2, \ldots$ be a sequence of iid r.v.s with mean 0 and let $S_n = X_1 + X_2 + \cdots + X_n$.

   (a) Show that $E[X_k|S_n] = \frac{S_n}{n}$ for $1 \leq k \leq n$. *Hint: Consider $E[S_n|S_n]$ and use linearity and symmetry.*

   (b) Use the preceding result to find $E[S_k|S_n]$ for $1 \leq k \leq n$.

2. In this problem, you will show that the continuous LoTP, first encountered in Section 7.1, is also a consequence of the Law of Total Expectation.

   (a) Let A be an event, let X be a continuous random variable, and let I_A be the indicator variable for A. Show that

   $$E[I_A|X = x] = P(A|X = x)$$

   using the definition of conditional expectation.

   (b) Let $g(x) = P(A|X = x)$. Use LOTUS to write $E[g(X)]$ as an integral.

   (c) Use the Law of Total Expectation, along with the Fundamental Bridge, to prove continuous LotP: for any event A and continuous r.v. X with PDF f_X,

   $$P(A) = \int_{-\infty}^{\infty} P(A|X = x) f_X(x)\, dx$$