

The Poisson Paradigm

1. For each of the following situations, determine whether it would be appropriate to use the Poisson Paradigm to approximate the variable N . If it is appropriate, briefly explain why the conditions for the paradigm are satisfied. If it isn't appropriate, explain what condition isn't satisfied, as well as why it might seem *tempting* to apply the Poisson Paradigm.
 - (a) Suppose $A_1, A_2, \dots$ is an infinite sequence of independent events, each with probability $p = 10^{-1000}$ of occurring. Let N be the number of events that occur.
 - (b) A particular data file is stored as a sequence of 10^6 binary digits. When the data file is copied, each term in the sequence has probability $p = 10^{-4}$ of having an error, independent of other terms. Let N be the number of errors in the sequence.
 - (c) Suppose two copies of each of 52 cards are thoroughly shuffled. Cards are drawn from the deck two at a time. Let N be the number of cards that are paired with their other copy.
 - (d) Let $X \sim \text{Bin}(100, 1/100)$, and for each $0 \leq n \leq 100$, let I_n be the indicator for the event " $X = n$ ". Let $N = I_0 + I_1 + I_2 + \dots + I_{100}$.
 - (e) Each of 23 students in a statistics classroom reveals the last 3 digits of their phone number. Let N be the number of phone numbers that are repeated at least once.
2. (*) Ten million people enter a certain lottery. For each person, the chance of winning is one in ten million, independently.
 - (a) Find a simple, good approximation for the PMF of the number of people who win the lottery.
 - (b) Congratulations! You won the lottery. However, there may be other winners. Assume now that the number of winners other than you is $W \sim \text{Pois}(1)$ and that if there is more than one winner, the prize is awarded to one randomly chosen winner. Given this information, find the probability that you win the prize (simplify).
 - (c) Suppose the prize is L . Find the overall expected value of the lottery in terms of L (i.e. do not assume that you win the prize).