Discrete Multivariate Distributions

1. Consider a pair of random variables X and Y whose joint PMF is specified by

$$p(x,y) = \frac{x}{2}\left(\frac{1}{2}\cdot 2^{-y}\right) + \frac{1-x}{2}\left(\frac{e^{-1}}{y!}\right) \qquad \text{for } x \in \{0,1\} \text{ and } y \in \{0,1,2,\dots\}$$

   (a) Show that p is indeed a valid jPMF.
   (b) Compute the marginal PMF for X. What well-known distribution is this?
   (c) Compute the marginal PMF for Y. Verify that this is **not** one of the named distributions we've seen.
   (d) Find the conditional distribution of Y given $X = x$. Based on the conditional PMF, is Y independent of X?
   (e) Create an intuitive story describing the relationship between X and Y.
   (f) Describe how you could use R to produce a sample of 1000 observations from the joint distribution of X and Y.

2. (*) Suppose X and Y are independent discrete random variables with $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. Show that if $Z = X + Y$, then $Z \sim \text{Pois}(\lambda_1 + \lambda_2)$.
   - Hint 1: Review AP2 from HW 4.
   - Hint 2: The binomial theorem states that for any real numbers a and b, $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$.