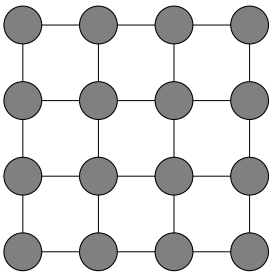
The Fundamental Bridge

1. Suppose you are watching cars drive by in a small town. Every minute, a car drives by, which is equally likely to be driven by any of the n residents of the town. What is the expected number of distinct residents you have seen after t minutes?

2. Suppose that positively and negatively charged particles are arranged in an $m \times n$ grid of the type shown here (in the $m = n = 4$ case).



   If there is a positively charged particle directly above, below, to the left, or to the right of a negatively charged particle, we say that those particles attract or are in an attracting pair (We do not consider or allow attraction along diagonals or at a distance greater than one). Note that one particle may be in multiple attracting pairs. Suppose that positive and negative particles are distributed on the grid mutually independently and uniformly randomly so that every node gets a particle. What is the expected number of attracting pairs in the $m \times n$ grid?

3. (*) Two researchers independently select simple random samples from a population of size N, with sample sizes m and n respectively (assume each researcher individually samples without replacement, but that together, the researchers may select the same person). Find the expected size of the overlap of the two samples in two ways:

   (a) Using the fundamental bridge.

   (b) By recognizing the number of people in the overlap as a particular named random variable.