Class Activity 10-24

Moments

1. Let $X \sim \text{Expo}(1)$. Compute the skew and kurtosis of X. Compare these values to those of the standard Normal distribution. What does this imply about the shape of the distribution of X?

2. (*) A distribution is called *symmetric unimodal* if it is symmetric (about some point) and has a unique mode. For example, any Normal distribution is symmetric unimodal. Let X have a continuous symmetric unimodal distribution for which the mean exists. Show that the mean, median and mode of X are all equal.

3. If X is symmetric, then there is a median of X equal to the mean of X, the skew of X is 0, and all odd central moments are 0. The converse, however, is not true in general (although it is a good rule of thumb for the types of distributions one encounters in practice).

   (a) Construct an **asymmetric** discrete random variable X with a *unique* median, where the median is equal to the mean of X. Plot the PMF.

   - (Challenge Problem): Repeat, but for a continuous variable X.

   (b) Construct an **asymmetric** discrete random variable X with mean and skew of 0. Plot the PMF.

   - (Challenge Problem): Repeat, but for a continuous variable X.

   (c) (Challenge Problem: Only work on this after completing all other problems) Let X and Y be independent random variables with the following densities:

   $$f_X(x) = \frac{1}{24}e^{-x^{1/4}} \qquad f_Y(x) = \frac{1}{24}e^{-x^{1/4}}[1 - \frac{1}{2}\sin(x^{1/4})]$$

   And let $Z \sim \text{Bern}(1/2)$, independent of X and Y. Define a variable W by

   $$W = ZX + (1 - Z)Y$$

   Show that W is antisymmetric, but that all odd central moments of W are 0.