The Exponential Distribution

1. The *Laplace Distribution* has PDF
$$f(x) = \frac{1}{2}e^{-|x|}$$
for all real x. The Laplace distribution is also called a *symmetrized Exponential* or *double Exponential* distribution, as evidenced by the following problems:

   (a) Use R to plot the PDFs for the Laplace and the Exponential(1) distribution, and explain how they relate.

   (b) Let $X \sim \text{Exp}(1)$ and let S be a random sign (i.e. $S \sim \text{DUnif}\{-1, 1\}$) independent of X. Let $Y = SX$. Find the PDF of Y and compare to the Laplace PDF.

   (c) Suppose Y has the Laplace distribution. Compute the **conditional** CDF and **conditional** PDF of Y given the event $Y \geq 0$. That is, find
$$F(t) = P(Y \leq t | Y \geq 0) \qquad \text{and} \qquad f(t) = \frac{d}{dt}F(t).$$
   How do the conditional CDF and conditional PDF compare to the CDF and PDF of an exponential random variable?

2. (*) Let T be the lifetime of a piece of hardware (how long before it breaks) and let T have CDF F and PDF f. The *hazard function* of T is defined by
$$h(t) = \frac{f(t)}{1 - F(t)}$$

   (a) Explain why h is called the hazard function and in particular, why $h(t)$ is the probability density for failure at time t, given that the hardware has survived up until then.

   (b) Show that an Exponential r.v has constant hazard function and conversely, if the hazard function of T is constant, then T must be $\text{Exp}(\lambda)$ for some λ.